DOCUMENT RESUME

ED 414 264 SP 037 668

AUTHOR	Clement, Lisa
TITLE	If They're Talking, They're Learning? Teachers' Interpretations of Meaningful Mathematical Discourse.
PUB DATE	1997-11-14
NOTE	16p.; Paper presented at the Annual Meeting of the American Educational Research Association (Chicago, IL, 1997).
PUB TYPE	Reports - Research (143) -- Speeches/Meeting Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Classroom Communication; Classroom Research; Communication Skills; *Discourse Modes; *Discussion (Teaching Technique); Elementary Education; Elementary School Mathematics; Higher Education; *Mathematics Education; Methods Courses; Preservice Teacher Education; *Problem Solving; Teacher Educators; Teaching Methods; *Thinking Skills
IDENTIFIERS	Preservice Teachers

ABSTRACT

In this study a university mathematics instructor was observed teaching a mathematics content course for preservice elementary teachers. The study focused on her interpretation of meaningful mathematical discussions. The instructor believed in the importance of teacher-student communication and discussion, but saw no differences between meaningful discussion in mathematics classrooms and discussions in other disciplines. After viewing the videotapes, she stated that good discussions had occurred because she had engaged students in discussion, questioned them, and probed for alternate solution strategies. However, she did not consider the nature of the discourse to be important in whether or not the discussion was meaningful. A student example indicates that students may never get the message that their thinking and understanding are important aspects of learning mathematics if they experience discourse that is more oriented toward operations than toward the underlying reasoning. A conclusion is that to promote meaningful discourse in mathematics classrooms, mathematics teachers, as researchers, must take the time to understand and explicate what meaningful mathematical discourse is and what it is not. (SM)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

Lisa Clement

ED 414 264

If They're Talking, They're Learning?
Teachers' Interpretations of Meaningful Mathematical Discourse[1]

Background

In 1991, the National Council of Teachers of Mathematics (NCTM) produced the *Professional Standards of Mathematics Teaching* — a document that called for alternative teaching and assessment methods. These alternative methods included group work, use of manipulatives, investigations, open-ended problems, whole class discussions, and an ability to orchestrate mathematical discourse. The intent of the document was to provide teachers with an image of those ways that they, as teachers, may facilitate their students mathematics learning. Some teachers have embraced the ideas in this document. For example, the teacher I selected to study had her students work in groups on open-ended problems and held whole class discussions. She was enthusiastic about utilizing many of the ideas mentioned in the document, and I wanted to investigate how this teacher, Sue was thinking about some of these ideas[2]. In particular, I was interested in her interpretations of meaningful mathematical discourse.

I observed Sue, a mathematics instructor at a local university, as she taught the first of four semester long mathematics content courses for pre-service elementary teachers. I interviewed her four times during the duration of the study. In two of the interviews I used stimulated recall techniques. In addition I analyzed videotapes of her teaching. Sue had earned a Bachelor of Science degree in mathematics and was working on her Master's degree in mathematics education at the time of the study. She was teaching from conceptually rich materials and had worked with the researchers who developed the materials for the course[3]. Sue had taught the course using the new materials one semester before I observed her teaching. She would often comment on the

[1]I would like to thank Jamal Bernhard for consenting to make student interview data available to me, and Melissa Mellissinos for her editing expertise. The assistance of both mathematics educators was invaluable to me.

[2]The names used in this paper are pseudonyms.

[3]I acknowledge that the terms *conceptually rich* and *worthwhile mathematical tasks* are particularly subjective. One may ask, "For whom are these materials conceptually rich and worthwhile?" I use the terms to mean that the materials themselves explicitly emphasize conceptual understanding, with little focus on procedures. I do recognize, however, that, as with any materials, the tasks can become proceduralized by both the instructors and the students.

SE 037 468

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

☐ This document has been reproduced as received from the person or organization originating it.

☐ Minor changes have been made to improve reproduction quality.

• Points of view or opinions stated in this document do not necessarily represent official OERI position or policy

PERMISSION TO REPRODUCE AND DISSEMINATE THIS MATERIAL HAS BEEN GRANTED BY

L. Clement

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

BEST COPY AVAILABLE

importance of communicating with her students. She appeared to have the preparation and interest one would need in order to utilize discourse as a way to have students come to understand mathematics, as well as materials to teach with that incorporated worthwhile mathematical tasks. I expected that Sue would promote rich discussion in her classroom. The two bulleted lists below summarize Sue's images and why I thought she would be an appropriate person to study.

Sue's image of a good mathematics teacher

- communicates at an appropriate level with all age groups (those just graduated from high school to those in their sixties);
- conveys an enjoyment of mathematics;
- has knowledge of mathematics; and
- understands that there is more than one way to solve problem.

Sue's image of a classroom setting in which mathematics learning was taking place

- teacher and students communicate in groups or whole class discussions;
- teacher explains well (as evidenced by student questions that build upon, rather than attempt to clarify, statements the teacher has made);
- students question the teacher; and
- teacher questions the students.

Sue's interpretations of meaningful mathematical discussions

Because of Sue's stated beliefs in the importance of communication and discussion, I further probed to specifically learn what she considered to be meaningful mathematical discourse. She was not familiar with the term *discourse*, so I asked her to consider meaningful mathematical communication or discussions instead. The two bulleted lists below provide a summary of Sue's interpretations of meaningful mathematical discussions and how she felt it was displayed in a classroom.

Sue's image of meaningful mathematical discussion

- listening on both sides (teacher and students);
- all parties (teacher-students or students-students) involved; and
- questioning rather than assuming meanings (important to ask, "What do you mean?").

Sue's image of a mathematics classroom situation in which meaningful mathematical discussions take place

- No difference between her image of a mathematics classroom and any other subject area classroom (nothing particular to the mathematics classroom);
- Listening, discussing, questioning each other;
- Exploring ideas, learning new words;
- Teachers ask and students answer non-rhetorical questions;
- Teacher engages in conversations with students;
- Teacher is able to listen to and restate students' utterances;
- Teacher waits for students to respond to her questions; and
- Teacher moderates most of the interactions, keeps students "going in a specific direction."

Transcripts and Commentary

I will use the following classroom transcript as an example of the ways in which Sue's images of meaningful discussions manifested themselves in the classroom. I will comment on the transcript in light of 1) Sue's reflections of it, 2) her images of meaningful discussions and 3) the sense her students might have made of it.

Sue asked her students about the following problem. The students had worked on it for homework the evening before coming to class:

> A $140,000 estate is to be split among 2 children and 2 grandchildren. The two grandchildren get the same amount of money and each child gets twice as much as each grandchild. How much does each child get? (Sowder, Sowder, Thompson, Thompson & Bowers, 1997)

Sue : *What'd you do? How'd you do it? ...*

Ann: *...We added 2x plus 2x plus x plus x., and...*

Sue: *All right, tell me where these (pointing to the "2x"s) came from.*

Ann: *Okay, we're told that um, ..., there's two children and two grandchildren and that the two grandchildren get the same amount of money, so we made that x, like that amount of money x, and...*

The student continued to give a report of the steps she had done to solve the problem. Sue then asked if anyone solved the problem in a different way. No students responded. She then asked, "Did everyone use variables to solve this problem? No? Then Tom, how'd you do it then?"

Tom: *At first I was really hung up on trying to divide things equally, and then I finally got it down to six, which for some reason, I don't know how, means you could look at the quantities in thirds and then I started to think about the groups that were involved and to me there were three groups involved, the two children made the 2 groups and the 2 grandchildren made one group, so I divided, I made a little diagram, and I split the box representing 140,000 into three groups and I split one of the groups into two to represent the two grandchildren, and then I just divided the amounts up of the 140,000 into two one-third segments and two one-sixth segments.*

Sue: *What kind of diagram did you draw, just a box?*

Sue asked Tom to describe the diagram he said he had made. She then asked, "The reason you did that was....Can you give me some insight as to why you decided to split it up into three groups?

Tom: *Well as I said at first I was thinking about splitting everything equally, in fact the first diagram I made had 4 groups and I thought, 'well that's not right' because I was plugging in $35,000 into each of those groups, and I knew that wouldn't work out, so then I started thinking, 'well maybe it's sixths', so I split the boxes into sixths, ... and that's when I started thinking, 'well, maybe it's a third,' and then um and then I was just trying to decide what quantities were involved and what groups, what are the relationships so that's when it sort of dawned on me that maybe one child represents a third of this, the second child the second third, and then that would let me split the last third into two pieces which then makes sense to the criteria of the problem.*

Sue drew a rectangle, cut it into three equal sized pieces, and wrote the word "child" in the first two pieces, cut the last third into two equal sized pieces and wrote, "grandchild" in both pieces.

Sue: *So you let this (pointing to the first rectangle) equal like one child, and this equal the other child, and then splitting this up in half means that one of these is one grandchild, and the other is the other one is the other grandchild. Good. And you ended up with the same numbers?*

Tom: *Yes.*

Sue: *Does that make sense what he did? That's very good. You could have also split this up into sixths though, right, it could have been done that way if you split this up into sixths but just realizing that the child will then get two of those and each grandchild only gets one, which is the same idea ,which is in effect what you were thinking because you know that one third is going to be twice one-half of a third and a half of a third is going to be one-sixth. Very good. Anybody else?"*

Sue's Reflection of the Lesson

Sue watched the videotape of the lesson and then spoke about her reactions to the discussion. She felt that the discussion between the two students and herself had been a good one. She liked the way that she continued to probe to get an alternate solution. She believed that Tom was able to explain his reasoning. She said that she wished that the discussion had involved more people.

Commentary

Sue's teaching and reflections resonate with her description of meaningful mathematical discussions and good mathematics teaching. She got students to state alternate solutions and involved students in the conversations. She asked non-rhetorical questions of her students and waited for her students to respond. She paraphrased Tom's explanation and then used Tom's diagram to present another alternate but similar solution. The classroom discourse was meaningful for Sue. In what ways was it meaningful for her students? Let's look at the transcript again:

Transcript with Commentary

The students asked Sue, the instructor, about the following problem:

A $140,000 estate is to be split among 2 children and 2 grandchildren. The two grandchildren get the same amount of money and each child gets twice as much as each grandchild. How much does each child get?

Sue: *What'd you do? How'd you do it?*

Notice that this question orients the students toward describing the operations performed in the problem and the calculations one did, rather than on how the students came to make sense of the problem.

One student discussed her solution. Sue then asked if anyone solved the problem in a different way. No students responded. Then she asked, "Did everyone use variables to solve this problem? No? Then Tom, how'd you do it then?"

Tom: At first I was really hung up on trying to divide things equally, and then I finally got it down to six, which for some reason, I don't know how, means you could look at the quantities in thirds, . . ."

Tom described his approach, but he continued to give a report of the steps that he had done, even after Sue asked for some insight into the problem. For a student in the class who had not yet made sense of the problem, it is not clear what sense he or she would have made by the end of the discussion. It might still not be apparent why Tom tried "sixths" and then "thirds" (even to Tom) and it also might not be apparent whether the other students in the class had made sense of his solution. The discussion centered around the calculations Tom did, not the reasoning behind the calculations. For example, Sue did not ask Tom about why the grandchildren together would make one group while each child would make a group. She never stated or had Tom (or any other class member) explicitly discuss the relative portions of money that the children and grandchildren received and why knowing something about the relative portions of money received may help students to think about the problem in thirds or sixths.

Sue further imputed to Tom a way of reasoning about the problem that Tom may or may not have considered when she said, "which is in effect what you were thinking." Sue finally paraphrased Tom's utterances, focusing on sixths and thirds, rather than relating the sixths and thirds back to the relative portions of money that the children and grandchildren received or insisting that her students focus on the aspects of the situation that invited Tom to think about the situation in sixths and thirds. When she offered a related but different solution to the problem, Sue discussed values (sixths and thirds) and calculations on the values (twice one-half of a third), without discussing the underlying reasoning. She stated, "You could have also split this up into sixths though, right, it could have been done that way if you split this up into sixths but just realizing that the child will then get two of those and each grandchild only gets one, which is the same idea ,which is in effect what you were thinking because you know that one-third is going to

be twice one-half of a third and a half of a third is going to be one-sixth. Very good. Anybody else?"

Commentary

During my interviews with Sue, she clearly stated the lack of need to distinguish between meaningful discourse in mathematics classrooms and classrooms from other disciplines. That is, Sue discussed only social norms and not sociomathematical norms (Yackel & Cobb, 1996). To Sue, as long as students were discussing *something*, they would come to understand deeply whatever subject they were learning. This view manifested itself in the mathematics classroom in at least two ways:

- her encouragement of students to present their solutions;
- her acceptance of reports of the steps students had done, rather than an insistence that students make explicit how they came to make sense of the problem and the reasoning behind the calculations they performed.

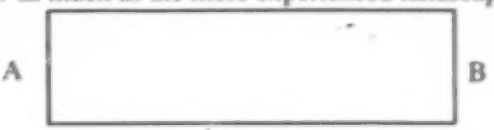
Sue did not distinguish between the reports of the calculations students performed and an explanation of the reasoning underlying the calculations. For Sue, the sheer acts of discussing and communicating were the salient features of meaningful discussions (i.e., the fact that students had the occasion to talk with one another or to the teacher), and not the content and nature of those discussions. This feature of her image of meaningful mathematical discourse emerged throughout the semester in ways similar to those describe above. Sue continued to involve her students in discussions, but the discussions entailed different students giving reports of the steps they had performed to solve problems. Implicit in this mode is the message to students that the salient features of a problem are the operations to perform on the numbers in the problem and not the reasoning underlying the calculations. Perhaps Sue had not thought explicitly about the way she had come to make sense of these problems. She was thus unable to consider that there may be aspects of meaningful discourse that are particular to mathematics classrooms. This lack of

consideration may have hindered her students from coming to understand the mathematics as deeply as they otherwise might have. Consider, for example, an interview with one of Sue's students.

Student example:[4]

The first section of the materials from which Sue taught included a strong emphasis on multiplicative reasoning, and the pre-service elementary teachers in these courses spent quite a bit of time with mathematical situations like the one given below. For example, one task very similar to this one that the materials cover and that the students discuss in class is how to cut up a candy bar between two people so that person A's part is a certain fraction of person B's part. The problem below was given on the class final. The student interviewed performed near the top of Sue's class, receiving the third-highest grade in the course.

Two landscapers mowed the lawn of a wealthy family. When they finished, landscaper A had mowed only 3/7 as much as the more experienced landscaper B mowed.



a) Mark on the drawing of the lawn to show how the mowing might have been done.

b) A's part is _____ times as much as B's part.

c) A's part is what part of the lawn?

d) What is the ratio of A's part to B's part?

e) If they are paid $150 for mowing the lawn, what would be a fair split of the $150?

The student's picture:

[4] I did not interview this student. Jamal Bernhard, a fellow graduate student, interviewed him, and then transcribed, and wrote the initial description of the student example. He consented to make this data available to me.

He answered all of parts (a) through (e) correctly.

One of the people working with the development of these materials was interested in collecting information on what the students were learning in these courses. Here is what transpired during a taped conversation after the final exam when the interviewer asked the student, Tim, about this item:

Tim: *Well the first thing I remember doing on the test is that I divided the space up into seven pieces, and then I started to think about shading three of those pieces, and I didn't think that that was the right way to approach it . . . I just thought, 'Well, maybe it's like one of those early problems where I found it easy to add the numerator and denominator, in this case the three and the seven.' So I split it up into ten sections. . . . I felt like it was better to do it that way than to divide it into 7 pieces and then shade off the 3.*

Int: *Okay. If you had divided it into 7 and then shaded 3 . . . why does this [pointing to Tim's picture] fit that problem better than what you talked about doing? Better than dividing it into 7?*

Tim: *[Long pause] You know, I think that, um, I just thought that, uh, that it was too simple just to think of it as three sevenths. I thought it would have to be a little more complicated than that, and that's why I went to trying to see if it worked out by dividing it into 10 pieces.*

Int: *Okay . . . So, is there a three-sevenths over in here, too [pointing to the picture]?*

Tim: *[Pauses] No.*

Int: *Okay, but you say that this [pointing to the picture] is a good picture to explain, 'Landscaper A had mowed only three sevenths as much as the more experienced Landscaper B.'*

Tim: *That's right.*

Int: *So can you explain how this picture demonstrates that?*

Tim: *Yeah, well, when I first, as I said – No, I can't (laughs). When I first drew it with 7 pieces, and then I shaded 3 pieces out of the 7, and then I looked down here at the other questions, I didn't think that I could answer the questions with that type of diagram. I thought it was more complicated than that, so then I went to this [pointing to the picture] and at that point I think I was confused about the problem and I decided to just quit while I was ahead.*

Int: *So you're saying that this diagram helped you answer these [pointing to questions (b) through (e)], or that it seemed to follow the rest of the problem, but you can't really explain why this drawing is a good representation of what's in the problem?*

Tim: *[Pauses] No, not really.*

Even though Tim answered the questions correctly and drew this picture in the same way that the interviewer would have, the interviewer was careful not to look at only what the student *did* and conclude that he and the student were necessarily making the same sense out of the situation or thinking about the problem in the same way. The interviewer also probed the student's thinking. Note that both the interviewer and Sue asked questions and had conversations with students. The surface features of the discourse were similar, yet the intent of the questions asked was notably different. Had the interviewer been satisfied with what the student *did*,, then we almost certainly would not know that the student did not understand.

We can think about Tim's understanding in light of the discourse in Sue's class—Sue oriented her discourse toward the calculations and procedures students *did* rather than on the reasoning underlying their calculations. The students may infer from this type of discourse that learning mathematics means stating procedures and getting right answers to problems (instead of

understanding that problem situations provide contexts in which to develop mathematical reasoning). We can observe this in the example above. Tim obtained a correct answer by following a particular procedure without having thought deeply about it. Tim and his classmates may never get the message that their thinking and understanding are important aspects of learning mathematics, particularly if, as in Sue's class, they experience discourse that is more oriented toward *what one does* rather than *how one thinks* .

Potential Implications for Students:

Consider some of the potential effects of the discourse like that which occurred in Sue's classroom over the course of a student's experience in mathematics classrooms:

- Students may continue to believe that the reason they are given problems is so that they can provide answers to them, and an "answer" is a number or a calculation.

- If students are asked only to "report" what they *did*, rather than how they were thinking, the salient features of a problem become the operations to perform on the numbers in the problem. Students may then come to believe that working problems means searching for operations to perform.

- Students may not be able to assess their own understanding of a situation if they are only interested in the solution.

Now imagine that these students will one day be the teachers of children:

Teachers may not have thought explicitly about the way they have made sense of problem situations. If teachers have not examined their own reasoning underlying calculations, then they will not have an image of what their students could gain by expecting them to make *their* reasoning explicit.

Consider in contrast, a teacher who has made her reasoning underlying calculations so explicit that she had a personal image of the type of thinking she would like her students to have.

This teacher could focus on aspects of situations that underlie the calculations performed rather than the calculations themselves. She might ask questions such as, "What occurred to you as you read this problem?" "What are some central relationships in this problem?" or simply "What is going on here?" These questions orient students' responses to how they were thinking about the situation, and not what they did.

Potential Implications for Students:

- Students may be more likely to view whole class discussions as occasions for students to reason and reflect on their reasoning.
- Students may give explanations grounded in the conceptions of the situation "automatically." That is, they may begin to think that a discussion of a problem *means* discussing the reasons underlying the calculations performed.
- Students may come to focus attention on the reasoning underlying the calculations, even when they are working on problems on their own. When they are thinking about problems, they may begin to try and make their own reasoning more explicit as they attempt to understand the problem.
- Students may come to expect to make sense of problems

Now again imagine that these students will one day be the teachers of children:

- Teachers may have made their reasoning underlying calculations so explicit that they have an image of the type of thinking they would like their students to have. The teachers could focus on aspects of situations that underlie the calculations performed rather than the calculations themselves.

Conclusion

In this study I examined one teacher's interpretation of meaningful mathematical discussions, and provided a classroom transcript to explicate how her interpretation manifested itself in the classroom. Sue stated that there were no differences between aspects of meaningful discussions in mathematics classrooms and those in other subject area classrooms. This belief helps to explain why, after viewing tapes of her teaching, Sue felt that good discussions had occurred—Sue and

her students were engaged in conversations; she questioned her students, and probed for students' alternate solutions strategies. These are all positive aspects of discussions in mathematics classrooms. For example, many students noted that Sue was one of the first instructors who had them consider alternate solution strategies. But because Sue did not consider the *nature* of the discourse to be an important feature of whether or not it was meaningful, her conversations with her students could potentially involve *any* aspects of the situation. In Sue's case, her conversations with her students focused on reports of what students *did*, and not explicitly about how students *thought*. Sue believed that as long as students and teachers were involved in conversations, then students would learn the subject matter deeply. Yet the ways in which one of the best students in the class understood the mathematics was by applying rules and procedures, and not by understanding the underlying concepts. It seems that having students talking and providing potentially worthwhile tasks did not help this student to come to understand the mathematics deeply. If we want to promote meaningful discourse in our mathematics classrooms (so that our students have greater opportunities to understand mathematics deeply), we need to take the time, as researchers, to understand and explicate what meaningful mathematical discourse is and what it is not.

In a broader context, the *NCTM Professional Standards of Mathematics Teaching* call for alternative teaching and assessment methods including group work, use of manipulatives, investigations, open-ended problems, whole class discussions, and an ability to orchestrate mathematical discourse. Some believe that this set of activities will necessarily lead to mathematical understanding for our students. Yet, in this study, whole class discussions did not appear to facilitate students' deep mathematical understanding. Other related questions for study are, "How does a teacher need to think about these activities in order to use them in ways that help students understand the mathematics deeply?" and "What are the salient features of these activities that will help our students to understand deeply the mathematics we teach?" Simply placing students in groups, having them use manipulatives, or in this case, having discussions with students, does not necessarily mean that we are creating opportunities for our students to learn mathematics deeply.

I am not stating that we can approach the problem by changing the way Sue talks. The problem is deeper than that. The way Sue talks is an indication of the way that she thinks about her talk and the ways in which she has come to understand and reflect on her own understanding of the mathematics. We will not be able to influence Sue's discourse without considering those components that underlie her spoken words in the classroom.

Reference

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: NCTM.

Sowder, J., Sowder, L., Thompson, P., Thompson, A., & Bowers, J. (1997). Quantitative Reasoning. San Diego.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. Journal for Research in Mathematics Education, 27(4), 458-477.